HOMEWORK DAY 4 – *Linear 1st order: Integrating factors. §1.4.1*

1. page 41: 1.

2. page 42: 2a.

3. page 42: 2c.

4. page 42: 3d.

5. page 42: 3f. (assume $t > 0$)

6. page 42: 4.

7. page 42: 5.

8. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object an the temperature of its surroundings. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time, and $k > 0$ is a positive parameter. Which of the following differential equations best models the temperature $H(t)$?

   ☐ $dH/dt = -kH$ ☐ $dH/dt = k(H - 70)$ ☐ $dH/dt = -k(H - 70)$

   ☐ $dH/dt = -k(70 - H)$ ☐ $dH/dt = -kH(H - 70)$

   Give 2 short and concise reasons for your choice.

9. page 34: 5.

10. page 32: 3. Use the solution to the cooling law $dT/dt = -k(T - T_e)$, $T(0) = T_0$ derived in class, $T(t) = T_e + (T_o - T_e)e^{-kt}$.

11. page 46: 1.

12. page 46: 4. You may use the formula $\int e^{at}\cos bt\,dt = \frac{e^{at}}{a^2+b^2}(a\cos(bt) + b\sin(bt))$. (Dont plot the function and find the time lag. They will be shown in the solutions.)

Extra Credit: Derive the formula given above using complex variables
$\cos bt = \frac{1}{2}(e^{ibt} + e^{-ibt})$, $\sin bt = \frac{1}{2i}(e^{ibt} - e^{-ibt})$.

HOMEWORK DAY 6 – *1st order ODEs: Autonomous equations $x' = f(x)$. §1.5.1*

13. For each of the following autonomous models $x' = f(x)$, (i) plot the growth rate x' vs x, (ii) sketch the phase line. (iii) State all equilibria and their stability (stable, unstable, semi-stable). (iv) plot a few solution curves $x(t)$ in the $t - x$ plane.

    (a) $dx/dt = 2x - 7$

    (b) $dx/dt = x^2(2 - x)$

(c) $dx/dt = x^2(5-x)^2(x-10)$

(d) $dx/dt = k - rx \ , \quad k > 0, r > 0$

(e) $dx/dt = x(x-8)^3$

(f) $dx/dt = 2x(1-x) - x/2$

(g) $dx/dt = ax - b$, $a > 0, b > 0$

(h) $dx/dt = \cosh(x) - 1$

Note: To draw the phase line, all you need to know is where $x' = 0$, $x' > 0$ or $x' < 0$. It is thus sufficient to know where $f(x)$ is positive or negative, rather than a full graph of f.

14. For the following problems determine the intervals where the growthrate is positive and where it is negative and use this result to plot the phase line. State all equilibria and their stability.

    (a) $dx/dt = (x-1)e^{3x}$

    (b) $dx/dt = (2-x)(1-e^{-2x})$

(c) $dR/dt = \dfrac{3R}{1+R^2} - 1$

15. Consider the autonomous DE

$$\frac{dx}{dt} = (x^2 - 36)(a - x)^2$$

where $a > 10$ is a constant.

(a) Draw the phase line (show your work). State and classify all equilibria

(b) Sketch an approximate graph of the solution curve $x(t)$ satisfying $x(0) = 8$. What is the limit of $x(t)$ as $t \to \infty$?