NAME: MATH 316 – HW 9 Due: Tuesday Nov 3, 2026

HOMEWORK DAY 27 – *Linear Algebra §4.2.1*

1. For the following linear systems of two equations in two unknowns x and y
   (i) sketch the graph of both equations
   (ii) Find all solutions (x, y) that satisfy the system and add those points, if any, to your graph in (i)
   (iii) Write the following systems in matrix form. $A\mathbf{x} = \mathbf{b}$ where $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$.
   (iv) Find the determinant of A, $\det(A)$.
   (v) If A is invertible, find the inverse A^{-1}.
   (vi) If A is invertible, find the unique solution of the linear system in (iii), using $\mathbf{x} = A^{-1}\mathbf{b}$. Compare your answer with the answer in (ii).

   (a) $x + 2y = 1$, $3x - y = 2$

(b) $x + 2y = 0$, $2x + 4y = 0$

(c) $x + 2y = 1,\ 2x + 4y = 2$

(d) $x + 2y = 1$, $2x + 4y = 0$

2. Find all solutions to the following systems

(a) $\begin{pmatrix} 0 & -3 \\ -2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(b) $\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(c) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(d) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

3. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 2 & 6 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, and λ be a constant. Simplify:

(a) $A + B =$

(b) $A\mathbf{x} =$

(c) $AB =$

(d) $A^{-1} =$

(e) $AA^{-1} =$

(f) $AI =$

(g) $A - I =$

(h) $2A =$

(i) $\lambda A =$

(j) $\lambda I =$

(k) $A - \lambda I =$

HOMEWORK DAY 30 – *Systems of 1st order linear ODEs*

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4. Let $A = \begin{pmatrix} 1 & 3 \\ \alpha & 4 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

   (a) For which values of α does $A\mathbf{x} = \mathbf{b}$ have a unique solution?

   (b) For which values of α does $A\mathbf{x} = \mathbf{0}$ have a infinitely many solutions?

   (c) For the value of α you found in (b), does $A\mathbf{x} = \mathbf{b}$ have a unique solutions, infinitely many solutions, no solution?

5. Let $A = \begin{pmatrix} 8 & -5 \\ 10 & -7 \end{pmatrix}$.

(a) Find the values of λ for which $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(b) For each value of λ you found in (a), find all solutions to $(A - \lambda I)\mathbf{x} = \mathbf{0}$.

6. Sketch the graphs of the following oriented curves. Always include orientation.

(a) $\mathbf{x}(t) = t\begin{pmatrix}1\\2\end{pmatrix}$, $t \in (-\infty, \infty)$

(b) $\mathbf{x}(t) = \begin{pmatrix}-1\\1\end{pmatrix} + t\begin{pmatrix}1\\2\end{pmatrix}$, $t \in (-\infty, \infty)$

(c) $\mathbf{x}(t) = e^{-t}\begin{pmatrix}1\\2\end{pmatrix}$, $t \in (-\infty, \infty)$

(d) $\mathbf{x}(t) = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $t \in (-\infty, \infty)$

(e) $\mathbf{x}(t) = \begin{pmatrix} 3\cos t \\ 5\sin t \end{pmatrix}$, $\quad t \in [0, 2\pi]$

(f) $\mathbf{x}(t) = e^{-2t} \begin{pmatrix} 3\cos t \\ 5\sin t \end{pmatrix}$, $\quad t \in (-\infty, \infty)$

7. Consider the system of two linear differential equations for $x(t)$ and $y(t)$,

$$x' = 2x - y \ , \quad y' = 3x - 2y \ .$$

(a) Write the system in matrix form $\mathbf{x}'(t) = A\mathbf{x}(t)$, where $\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

(b) Show that the function $\mathbf{x}(t) = e^{-t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ solves the equation in (a).

(c) Sketch a graph of the function in (b).

8. Consider the system of two linear differential equations for $x(t)$ and $y(t)$,

$$x' = 6x + 3y \ , \quad y' = 2x + y \ .$$

(a) Write the system in matrix form $\mathbf{x}'(t) = A\mathbf{x}(t)$, where $\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

(b) Show that the function $\mathbf{x}(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + e^{7t} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ solves the equation in (a).

(c) Sketch a graph of the function in (b).

9. Consider the system of two linear differential equations for $x(t)$ and $y(t)$,

$$x' = y \ , \quad y' = -x \ .$$

(a) Write the system in matrix form $\mathbf{x}'(t) = A\mathbf{x}(t)$, where $\mathbf{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

(b) Show that the function $\mathbf{x}(t) = \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix}$ solves the equation in (a).

(c) Sketch a graph of the function in (b).

HOMEWORK DAY 31 – *Solving systems. Initial values problems.*

10. Consider the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \mathbf{x}_o$ given by $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

    (a) Find the eigenvalues and eigenvectors of A

(b) State the general solution.

(c) Solve the initial value problem.

11. Consider the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \mathbf{x}_o$ given by $\mathbf{x}' = \begin{pmatrix} -5 & 4 \\ 1 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of A

(b) State the general solution.

(c) Solve the initial value problem.