NAME: MATH 316 – HW 10 Due: Tuesday Nov 10, 2026

HOMEWORK DAY 32 – *Solving systems, real eigenvalues, saddle. §4.4.1*

1. Consider the system from HW 9 (Problem 10)

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$$

(a) State the general solution (copy over from HW 9, #10).

(b) Draw the phase portrait. Classify the equilibrium.

(c) State the solution that satisfies $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ (copy over from HW 9, #10).

Highlight the graph of this solution curve in your phase portrait in (b).

2. Consider the system from HW 9 (Problem 11)

$$\mathbf{x}' = \begin{pmatrix} -5 & 4 \\ 1 & -2 \end{pmatrix} \mathbf{x}$$

(a) State the general solution (copy over from HW 9, #11).

(b) Draw the phase portrait. Classify the equilibrium.

(c) State the solution that satisfies $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ (copy over from HW 9, #11).

Highlight the graph of this solution curve in your phase portrait in (b).

3. Consider the system of differential equations

$$\mathbf{x}' = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} \mathbf{x}$$

(a) Find the general solution.

(b) Draw the phase portrait, showing a representative set of solution curves in the phase-plane. Classify the equilibrium.

(c) Find the unique solution that satisfies the initial condition $\mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and highlight that solution curve in your phase plane above.

Highlight the graph of this solution curve in your phase portrait in (b).

4. Consider the initial value problem $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

   (a) Find the general solution.

(b) Plot the phase portrait and classify the equilibrium.

(c) Solve the initial value problem and highlight the curve in your phase plane above.

5. p217: (Given e-vals and e-vecs, write general solution and categorize equilibrium.)
   1(a)

   1(b)

   1(c)

   1(d)

6. Consider the initial value problem $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

   (a) Find the general solutions.

(b) Plot the phase portrait and classify the equilibrium.

(c) Solve the initial value problem and highlight the curve in your phase plane above.

7. Consider the system $\mathbf{x}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{x}$

   (a) Find the eigenvalues, classify the equilibrium, and sketch the phaseportrait. Include the proper orientation in all cases.

(b) Find the general solution.

(c) Find the solution that solves the initial condition $\mathbf{x}(0) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and add its graph in the phase portrait in (a)

8. Consider the system $\mathbf{x}' = \begin{pmatrix} 3 & 2 \\ -4 & -1 \end{pmatrix} \mathbf{x}$

   (a) Find the eigenvalues, classify the equilibrium, and sketch the phaseportrait. Include the proper orientation in all cases.

(b) Find the general solution.

(c) Find the solution that solves the initial condition $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and add its graph in the phase portrait in (a)

9. Consider the system $\mathbf{x}' = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} \mathbf{x}$

   (a) Find the eigenvalues, classify the equilibrium, and sketch the phaseportrait. Include the proper orientation in all cases.

(b) Find the general solution.

(c) Find the solution that solves the initial condition $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and add its graph in the phase portrait in (a)

10. Consider the system $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x}$

(a) Find the eigenvalues, classify the equilibrium, and sketch the phaseportrait. Include the proper orientation in all cases.

(b) Find the general solution.

(c) Find the solution that solves the initial condition $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and add its graph in the phase portrait in (a)

HOMEWORK DAY 35 – *Solving systems, repeated eigenvalues. §4.4.3*

11. For the following systems of differential equations

    - Find the general solution
    - Plot the phase portrait. Classify the equilibrium.

    (a) $\mathbf{x}' = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}$

12. In this problem, the coefficient matrix contains a parameters, α.

$$\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x}$$

(a) Determine the eigenvalues in terms of α.

(b) Find the critical value or values of α where the qualitative nature of the phase portrait for the system changes.

(c) Sketch a phase portrait for a value of α slightly above, and another slightly below, each critical value.