Math 316 REVIEW Name: Solutions

The following list of problems cover basic material from Pre-Calculus, Trigonometry, Calculus I and II of which you are expected to have a working knowledge. Please do them all and review material that you feel that you are weak on. If you had to struggle with many of these questions, I would suggest retaking a lower level class in order to fill in the gaps.

1.) Differentiate the following functions:

$\frac{d}{dx}(e^u) = e^u du$

$g(x) = xe^{-kx^2}$, k is a constant

$g'(x) = e^{-kx^2} + x(-2kxe^{-kx^2})$

$= e^{-kx^2}(1 - 2kx^2)$

$g(x) = \int_0^x \tan^{-1}(e^s)ds$

$g'(x) = \tan^{-1}(e^x) + \int_0^x \frac{\partial}{\partial x}(\tan^{-1}(e^s))ds$

$= \tan^{-1}(e^x)$

$g(y) = \sqrt{y^3 + 1}$. Express your answer using radical notation.

$g'(y) = \frac{1}{2}(y^3+1)^{-1/2} \cdot (3y^2) = \dfrac{3y^2}{2\sqrt{y^3+1}}$

$g(t) = (t - \pi)\sin(2\pi(t - 10))$

$g'(t) = \sin(2\pi(t-10)) + (t-\pi)2\pi\cos(2\pi(t-10))$

2.) Evaluate the following limits:

$$\lim_{x\to\infty} \frac{e^{2x} - 3e^{3x}}{-4e^{2x} + e^{3x}}$$ $\rightarrow$ indeterminant form, $\frac{-\infty}{\infty}$, multiply by $\frac{e^{-3x}}{e^{-3x}}$

$$= \lim_{x\to\infty} \frac{e^{-x} - 3}{-4e^{-x} + 1}$$

$$= \frac{0 - 3}{-4(0) + 1} = -3$$

$$\lim_{x\to-\infty} \frac{e^{2x} - 3e^{3x}}{-4e^{2x} + e^{3x}}$$ $\rightarrow$ indeterminant form, $\frac{0}{0}$, multiply by $\frac{e^{-2x}}{e^{-2x}}$

$$= \lim_{x\to-\infty} \frac{1 - e^{x}}{-4 + e^{x}} = \frac{1 - 0}{-4 + 0} = \frac{-1}{4}$$

$$\lim_{x\to 0^+} x \ln x$$ $\rightarrow$ indeterminant form, $0 \cdot (-\infty)$,

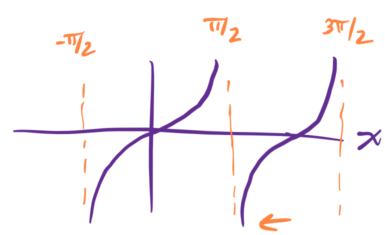
$$\lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-1/x^2} = \lim_{x\to 0^+} -x = 0.$$

(L'Hôspital)

$$\lim_{x\to 0} \frac{x}{e^{-x}} = \frac{0}{1} = 0.$$

$$\lim_{x\to \frac{\pi}{2}^+} \tan x = -\infty$$

Graph of tan(x):



$$\lim_{x\to\infty} \frac{\cos x}{x}.$$ Note $-1 \le \cos x \le 1$, so $\frac{-1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$.

by the squeeze theorem, $\lim_{x\to\infty} \frac{-1}{x} = \lim_{x\to\infty} \frac{1}{x} = 0$, so $\lim_{x\to\infty} \frac{\cos x}{x} = 0$

3.) Evaluate the following integrals.

$$\int \frac{x+2}{(x-1)(x+3)}dx$$

$$= \frac{3}{4}\int \frac{1}{x-1}dx + \frac{1}{4}\int \frac{1}{x+3}dx$$

$$= \frac{3}{4}\ln|x-1| + \frac{1}{4}\ln|x+3| + C$$

PFD $\longrightarrow$ $\frac{x+2}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$\Rightarrow A(x+3) + B(x-1) = x+2$

$\Rightarrow (A+B)x + (3A-B) = x+2$

$\Rightarrow A+B=1, \quad 3A-B=2$

$B = 1-A \Rightarrow 3A-1+A=2$

$4A=3 \Rightarrow A = 3/4$

$\Rightarrow B = 1/4$

$$\int \tan^{-1} x dx = x\tan^{-1}x - \int \frac{x}{x^2+1}dx$$

$u = \tan^{-1}x \qquad dv = dx$

$du = \frac{1}{x^2+1} \qquad v = x$

$u = x^2+1$

$du = 2x\,dx$

$$= x\tan^{-1}x - \frac{1}{2}\int \frac{du}{u}$$

$$= x\tan^{-1}x - \frac{1}{2}\ln(x^2+1) + C$$

$$\int \sin^2 2s ds = \int \frac{1-\cos 4s}{2}ds = \frac{s}{2} - \frac{1}{2}\int \cos 4s\,ds$$

$u = 4s, \quad du = 4ds$

$$= \frac{s}{2} - \frac{1}{8}\sin 4s + C$$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\frac{x^3}{3}$

$$\int x\sqrt{3x^2+1}dx = \frac{1}{6}\int u^{1/2}du = \frac{2}{18}(3x^2+1)^{3/2} + C$$

$u = 3x^2+1$

$du = 6x\,dx$

4.) Algebra Problems and Trigonometry

Solve for x: $e^{2x}+5e^{x}-5=1$ $\quad u=e^{x}$

$u^2+5u-6=0$

$(u+6)(u-1)=0 \Rightarrow \cancel{u=-6}\quad u=1 \Rightarrow e^{x}=1$

$\Rightarrow \boxed{x=0}$

(e^x cannot be neg.)

Solve for x: $\ln x+\ln(x+1)=\ln 6$

$\Rightarrow \ln(x^2+x)=\ln 6$

$x^2+x=6 \Rightarrow x^2+x-6=0$

$(x+3)(x-2)=0 \Rightarrow \cancel{x=-3},\ \boxed{x=2}$

(can't take $\ln(-2)$)

Express $y=2x^2+6x+1$ in the form $y=a(x-h)^2+k$

$y=2(x^2+3x)+1$

$=2\left(x^2+3x+\frac{9}{4}\right)+1-\frac{9}{2}$

$=2\left(x-\frac{-3}{2}\right)^2-7/2$

Write $\sqrt{(\sqrt{x})^3}$ in exponential form.

$\sqrt{x^{3/2}}=x^{3/4}$

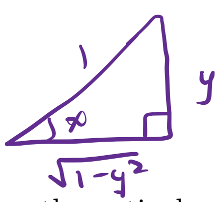
Solve the system of equations:

$2x+3y=1$

$-x-\frac{3}{2}y=2$ $\quad -\frac{3}{2}y-2=x \Rightarrow -3y-4+3y=1 \Rightarrow -4=1$

$\boxed{\text{No solution}}$

Find the value of $\tan\left(\sin^{-1}y\right)$ $=\tan(x)$



$x=\sin^{-1}y$

$\sin x=\frac{y}{1}$

$\Rightarrow \tan(\sin^{-1}y)=\frac{y}{\sqrt{1-y^2}}$

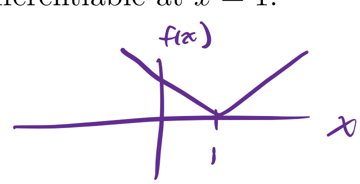
Use correct mathematical notation to describe what it means to say that a function $f(x)$ is differentiable at $x=a$.

$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ exists

or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists

Provide an example of a function that is not differentiable at $x=1$.

$f(x)=|x-1|$

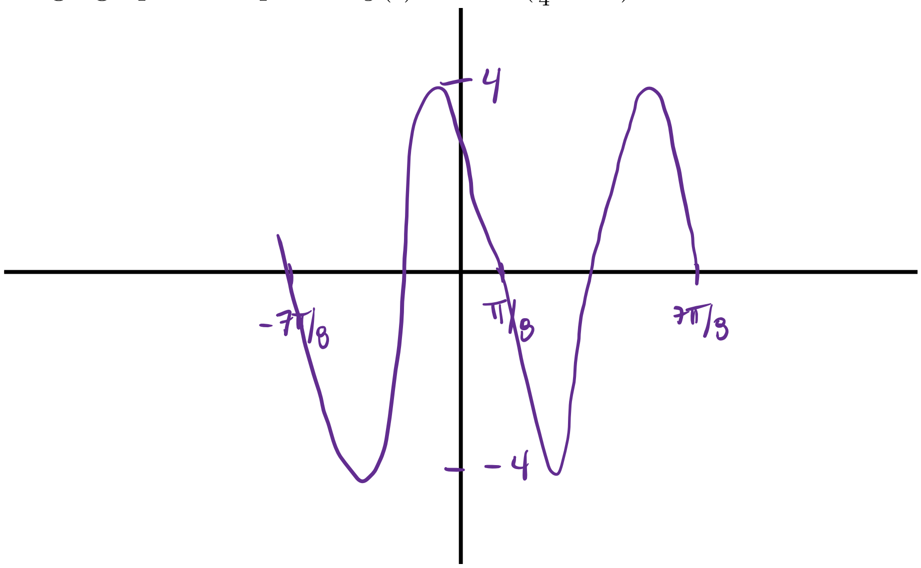


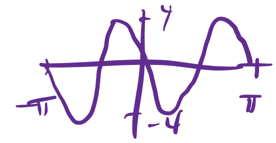
Provide a rough graph the equation $g(t) = 4\sin\left(\frac{\pi}{4} - 2t\right)$. Neatness counts!

$4\sin\left(-2\left(t - \frac{\pi}{8}\right)\right)$

shifted $4\sin(-2t)$ by $-\frac{\pi}{8}$

$4\sin(-2t) = -4\sin(2t)$





Provide a rough graph the equation $g(x) = \dfrac{3x^2+2}{x+1}$. Neatness counts!

$$\begin{array}{r} 3x-3 \\ x+1 \,\big)\, 3x^2+2 \\ -(3x^2+3x) \\ \hline 2-3x \\ -(-3-3x) \\ \hline 5 \end{array}$$

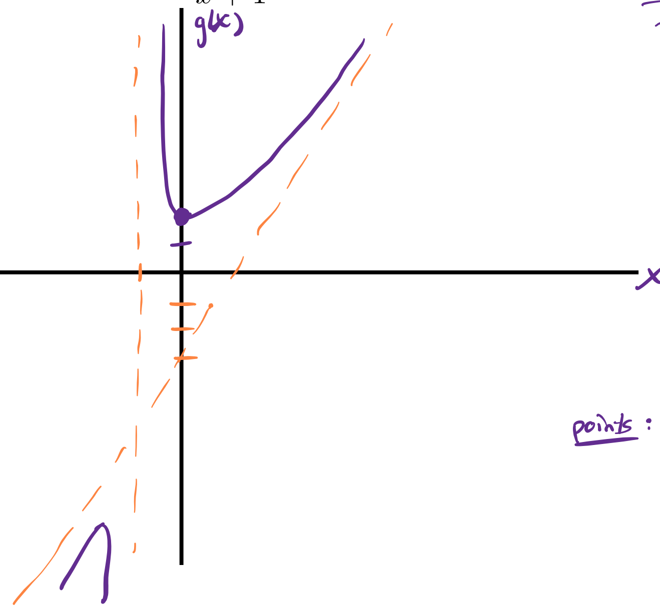
$g(x) = 3x - 3 + \dfrac{5}{x+1}$

vertical asym: $x = -1$

oblique asym: $y = 3x - 3$

points: $x=0$: $g(x) = 2$

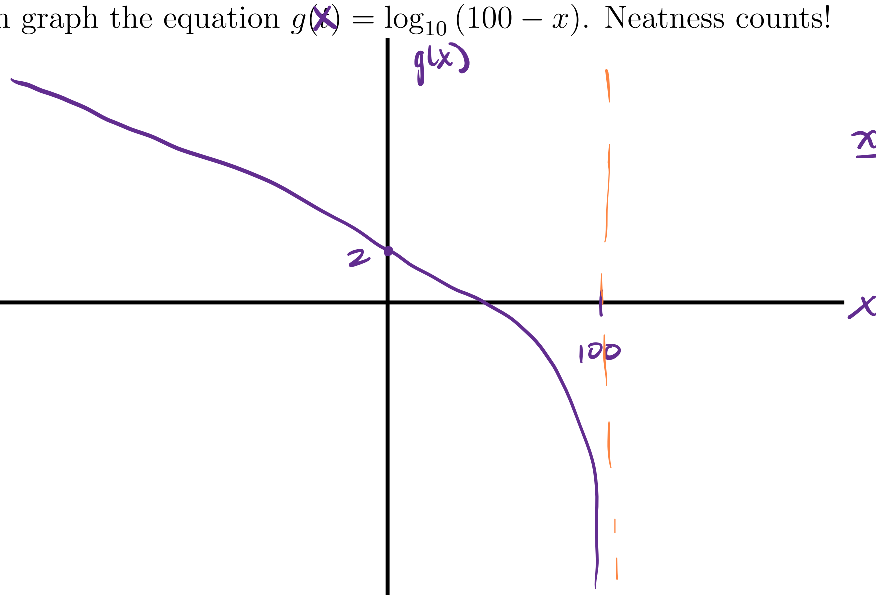
$x = -2$ $g(x) = \pm$



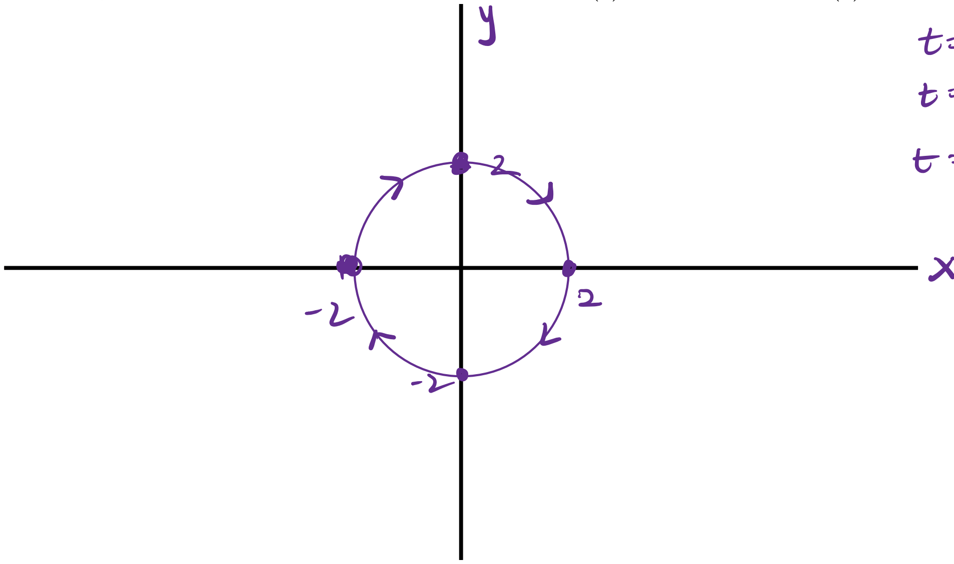
Provide a rough graph the equation $g(x) = \log_{10}(100 - x)$. Neatness counts!

reflection + shift

$x=0$: $\log_{10}(100) = 2$



Provide a rough graph of the parametric equations $x(t) = 2\sin t$ and $y(t) = 2\cos t$. Neatness counts!

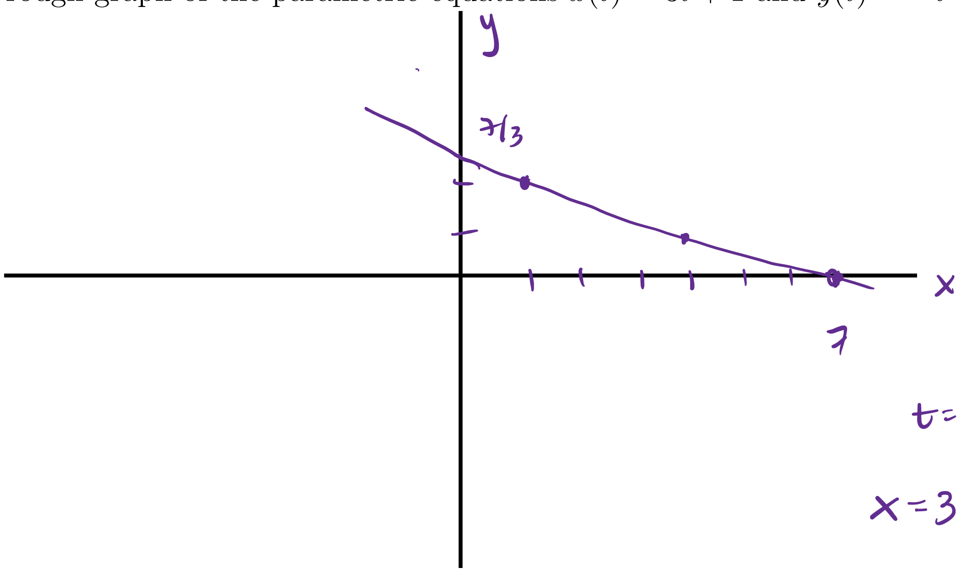


$t=0: (0,2)$

$t=\pi: (0,-2)$

$t=\frac{\pi}{2}: (2,0)$

Provide a rough graph of the parametric equations $x(t) = 3t + 1$ and $y(t) = -t + 2$. Neatness counts!



$t=0: (1,2)$

$t=1: (4,1)$

$t=2: (7,0)$

$t = 2-y$

$x = 3(2-y)+1$

$3y = -x+7$

$y = -\frac{x}{3} + \frac{7}{3} = \frac{-1}{3}(x-7)$

slope: $\frac{-1}{3}$

5.) Evaluate the following improper integrals.

$$\int_0^{\infty} e^{-x}dx = \lim_{b\to\infty} \int_0^b e^{-x}\,dx = \lim_{b\to\infty} -e^{-x}\Big|_0^b$$

$$= \lim_{b\to\infty} \left(-e^{-b} - (-1)\right) = 1$$

$$\int_0^1 \frac{1}{\sqrt{x}}dx = \lim_{a\to 0} \int_a^1 x^{-1/2}\,dx = \lim_{a\to 0} 2x^{1/2}\Big|_a^1$$

$$= \lim_{a\to 0} \left(2 - 2\sqrt{a}\right) = 2$$

$$\int_1^{\infty} \frac{1}{\sqrt{x}}dx = \lim_{b\to\infty} \int_1^b x^{-1/2}\,dx = \lim_{b\to\infty} 2x^{1/2}\Big|_1^b$$

$$= \lim_{b\to\infty} \left(2\sqrt{b} - 2\right)$$

= DNE, diverges