MATH 316 Review for Exam 1

A range or sample textbook problems is given (which include several homework problems). Choose a subset of these and the homework to review the material.

TOPIC : Solutions of first order ODEs $\dfrac{dx}{dt} = f(x,t)$

- **Solutions to ODE.** Show that a given function solves a given ODE.
  Examples: §1.1.2 (p9-10): 1-9

- **Exponential Growth/Decay.**
  Examples: §1.1.2 (p9-10): 12,13,15,16,17.

- **Direction fields.** Given a direction field for an ODE, draw several solution curves. Draw the solution that satisfies a given initial condition.

- **Antiderivatives.** Be able to write an antiderivative as a definite integral with variable bound. Properly use the Fundamental Theorem of Calculus.
  Examples: §1.2 (p19-21): 10,11.

- **Separable equations.** Solve separable equations

  $$\frac{dx}{dt} = f(x)g(t)$$

  using separation of variables. Careful!! Only separate variables if $f(x) \neq 0$. Also: be careful with your algebra, and always check your antiderivatives.
  Examples: §1.3 (p26-29): 1-8,19, $y' = r(a - y)$

- **Linear equations.** Solve linear equations

  $$x' + p(t)x = q(t)$$

  using integrating factors.
  Explain why the integrating factor μ equals $\mu(t) = e^{\int p(t)dt}$.
  Examples: §1.4 (p 42): 2-4

- **Autonomous equations.** For autonomous systems

  $$\frac{dx}{dt} = f(x)$$

  Find equlibria.
  Draw x' vs x and use it to draw phaseline.
  Use phaseline to find stability of all equilibria.
  Given an initial condition $x(t_0) = x_0$, find the limiting behaviour of $x(t)$ as t increases.
  Use phaseline to draw solutions curves. (Skip Theorem 1.32)
  Examples: §1.5 (p63): 2-3.
  True/False: All autonomous 1st order ODEs are separable. If true, give illustrating example. If false, give counterexample.
  True/False: All separable 1st order ODEs are autonomous. If true, give illustrating example. If false, give counterexample.

- **Modeling.** Given a simple physical rule, write down the corresponding equation.
  Examples: Newton's law of cooling. Mixing. §1.3.1 (p20): 27. §1.3.2 (p32): 1-4. §1.3.3 (p34): 2.

- **Population models.** State the equations for simple population models: exponential growth, exponential growth with harvest, logistic growth, logistic growth with harvest.

- **Dimensions.** State the dimension of all variables and parameters in an equation in terms of base dimensions (length, time, mass, temperature).

- **Theory** What can you say about the existence and uniqueness of a solution to a nonlinear first order IVP, $x' = f(t, x), x(t_0) = x_0$?
  Given an ODE, find all initial conditions for which Theorem 1.38 guarantees existence of a unique solution.

  What can you say about the existence and uniqueness of a solution to the linear IVP: $x' + p(t)x = q(t), x(t_0) = x_0$?

- **Combo problem**
  Consider the temperature T of coffee in a cup in a room of ambient temperature T_a.
  (a) Using Newton's law of cooling, find an (autonomous) ODE modeling the temperature T.
  (b) Draw the phaseline, determine the stability of all equilibria, and draw several solution curves in the t-T plane.
  (c) Find *all* the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of separation of variables.
  (d) Find *all* the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of integrating factor.
  (e) For how long does the temperatures function $T(t)$ exist, continuously? Explain, using both your above results and the theoretical results of §2.4.
  (f) If the coffee in two cups in the same room have two different temperatures at time t_0, will they ever have the same temperature? Explain.