

MATH 316 — REVIEW Exam 3

Laplace Transform

For all problems, use the Table of Transforms given on our website.

A Using the definition of the Laplace Transform

Examples: Find Laplace Transform using the definition

- (a) Derive formulas 7, 8, 9a, 11 in the Table.
- (b) Using the definition, find $\mathcal{L}[t]$.
- (c) Using the definition, find $\mathcal{L}[x']$ (entry in table). Using this result find $\mathcal{L}[x'']$ (entry in table).
- (d) Use the definition to find the transform of functions defined piecewise (Example: HW 6, #1)

B Compute Laplace Transform and inverse Transform using the table.

In the exam, you will be given the table posted on the course website. You may be asked to state which entry in the table you are using, including, if applicable, the values of a , $f(t)$, $F(s)$.

Examples: Find the transforms of

- (a) $f(t) = t^2 - \cos(5t)$
- (b) $f(t) = e^{3t} \sin 2t$
- (c) $f(t) = te^{-4t}$
- (d) HW 6, #2.
- (e) $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ -2, & 3 \leq t \end{cases}$
- (f) $f(t) = H(t - \pi) \sin(t - \pi)$
- (g) $f(t) = H(t - 2)t^2$
- (h) $f(t) = e^t H(t - c)$
- (i) $f(t) = \delta(t - 3)$

Examples: Find the inverse transforms of

- (a) $F(s) = \frac{3}{s^2 + 4}$
- (b) $F(s) = \frac{4}{(s - 1)^3}$
- (c) $F(s) = \frac{3s}{s^2 - s - 6}$
- (d) $F(s) = \frac{3s}{s^2 + 5s + 6}$
- (e) $F(s) = \frac{2s + 2}{s^2 + 2s + 5}$
- (f) $F(s) = \frac{2s}{s^2 + 4s + 10}$
- (g) $F(s) = \frac{7}{s} e^{-4s}$

- (h) $F(s) = \frac{1}{s(s-2)} e^{-s}$
- (i) $F(s) = \frac{7s+1}{s^2+9} e^{-\pi s}$
- (j) $F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$
- (k) $F(s) = \frac{s}{(s+1)(s^2+4)}$ (your answer may not be in terms of a convolution integral)

C Solve initial value problems with the Laplace Transform.

Examples

- (a) $x'' - x' - 6x = 0, \quad x(0) = 1, \quad x'(0) = -1$
- (b) $x'' - 4x' + 5x = 0, \quad x(0) = 1, \quad x'(0) = 0$
- (c) $x'' - 2x' + 2x = e^{-t}, \quad x(0) = 0, \quad x'(0) = 1$
- (d) $x'' + \omega^2 x = \cos(2t), \quad x(0) = 1, \quad x'(0) = -1, \quad \omega \neq 2$
- (e) $x'' + 4x = t$ if $0 \leq t < 1$, 0 if $t \geq 1$
- (f) $x'' + x' + \frac{5}{4}x = \sin t$ if $0 \leq t < \pi$, 0 if $t \geq \pi$
- (g) $x' + 5x = H(t-2), \quad x(0) = 1$
- (h) $x'' + 3x' + 2x = \cos(\alpha t), x(0) = 1, x'(0) = 0$ (your answer may be in terms of the convolution integral)
- (i) $x'' + x' + \frac{5}{4}x = 1 - H(t-\pi), x(0) = 1, x'(0) = -1$ (your answer may be in terms of the convolution integral)
- (j) Homework Problems

D Sketch a graph of

- (a) Functions defined piecewise.
- (b) $f(t) = t - (t-2)H(t-2)$ (rewrite as function defined piecewise)
- (c) HW 8 #2, 4
- (d) $f(t) = 3e^{-t/2} \sin(4t)$
- (e) $f(t) = 2t \cos(t - \pi/4)$